

Perspective on Mathematical Modeling



By Gary De Young

Preface

This article reflects my thoughts concerning a Christian approach to mathematical modeling. Often within the community of mathematical modelers, little thought is given to the interplay of faith and modeling. Typically the focus of a modeler is on solving a problem or answering a question. From a purely mathematical viewpoint, making a model is thought of as an intellectual exercise with few ramifications. From an applied mathematical point of view, it is realized that models may have far-reaching effects if they are adopted and used. My first purpose in writing this article is to show that for both cases (an intellectual exercise or an

applied mathematical model), it is important for a Christian to understand the assumptions and limitations of mathematical modeling. My second purpose is to present an article that is accessible to undergraduates interested in mathematical modeling so that they can learn not only about the basis of modeling but also about the interaction of faith and modeling.

Introduction

In the novel *Polar Shift*,¹ authors Clive Cussler and Paul Kemprecos build an exciting adventure around a fictitious set of theorems by Kovac. These theorems provide the scientific basis for extreme manipulation of natural phenomena. Examples include inducing rogue ocean waves and massive whirlpools in the open ocean as well as reversing the polar magnetic fields. The book mixes popular scientific ideas with imagination and computer simulations to produce a highly entertaining thriller novel.

Implied in this stimulating book is a mathematical model that describes a natural resonance phenomenon. Through computer simulations using theorems and scientific constructs, the villain acquires the ability to manipulate forces within nature that are normally thought to be outside the influence of mankind. Even the heroes of this story rely on theorems and computer simulations to save the day. This idea is not as far-fetched as it may sound; for example, electromagnetic waves were discovered through mathematical equations, and the use of electromagnetic waves has transformed the world over the last century and a half, according to Morris

Dr. Gary De Young is Professor of Mathematics at Dordt College.

Kline.² What I find of particular interest is the idea that the knowledge not only of mathematical theorems but also of computer simulation of mathematical models implies power.

Using mathematics to understand and direct nature is not a new idea; it has proved very successful in areas of physics, engineering, chemistry, and other “hard” sciences. More recently, mathematical models have been applied to life sciences, biology, economics, and environmental science, etc. With the rise and availability of increasing computing power, computerized simulations are extending the reach of mathematical models. The subject I wish to explore is the process and basic assumptions of mathematical modeling.

What is mathematical modeling? It is using mathematics to understand some aspect of a non-mathematical entity. For illustrative purposes we will refer to the non-mathematical entities as things in the “physical world.” The term “physical world” should not be limited to the world of atoms. For our purposes, it also includes other non-mathematical entities, such as social phenomena. The “mathematical world” is where mathematical results are derived. Rather than get bogged down in what or where these worlds are, we will assume an intuitive understanding of what is meant by “mathematical world” and “physical world.”

2 Foundations

The thrust of mathematical modeling is to use mathematics to understand the physical world; however, we begin by considering how the physical world informs mathematical knowledge. This connection is important since (1) it illustrates the basis for the certainty of mathematical knowledge, (2) it illustrates the interplay between the mathematical world and the physical world, and (3) it provides a basis for the correspondence between the mathematical world and physical world. All are important for mathematical modeling.

2.1 Mathematics and Certainty

One often-quoted reason for studying mathematics is its certainty. But how does mathematics increase our plan’s certainty level? We will answer this question by considering the concept of consistency. If a system is inconsistent, its level of certainty is greatly reduced. We shall see that the certainty within a mathematical system

is informed by physical models, the main point being that mathematical certainty should be based on God’s providential and sustaining hand in creation.

A mathematical system is consistent if there are no contradictions possible within it. That is, no conceivable statement in the system can be shown to be both true and false at the same time. If a contradiction or paradox³ appears, the underlying assumptions of the system (axioms) are re-evaluated, leading either to an explanation of the paradox or a modification to eliminate the contradiction. Contradictions in a mathematical system are disastrous. If a contradiction appears, all results collapse like a house of cards!

How can one be 100 percent certain that contradictions will not appear? It turns out that we can never be completely certain. After all, showing the veracity of all possible derived statements, even those not yet conceived, is a tall order. However, using physical models, we can gain assurances of system consistency without knowing all possible statements.

I will illustrate this point with three-point geometry. The axioms of three-point geometry are as follows:

A1: There exist exactly three points.

A2: Any two distinct points are contained in exactly one line.

A3: No line contains all points.

A4: Any two distinct lines contain at least one point in common.

A theorem, or truth, in this system is the statement that “*Two distinct lines contain exactly one point in common.*” This statement differs from axiom A4, since axiom A4 allows more than one point in common. (Note the word *exactly* in the theorem.) Here is the reasoning that establishes the theorem: By axiom A4, there must be at least one common point contained in both lines, so there cannot be distinct lines with no points in common. Suppose that there is more than one common point contained on both lines. Then the two lines must have at least two points in common. According to axiom A2, those two points determine exactly one line. So our “lines” must be a single line. This finding contradicts the assumption that we started with two distinct lines, so there cannot be more than one point in common.

Consider another statement: “*There are exactly*

three lines.” This truth depends on all four axioms, and its argument is more complicated. Statements that can be proved true from the axioms of the system are called theorems. A collection of a method of reasoning, of axioms, and of provable theorems is called an axiomatic system. Axiomatic systems are the organizational standard for mathematical knowledge. An axiomatic system, and thus mathematical knowledge, can have realizations.

Perhaps in your mind you envisioned, or better yet attempted to draw, a picture of the axioms of

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three-point geometry when you first read them. A common one, where all the axioms hold, is that of a triangle. The points are vertices, and connections between vertices are the edges of the triangle. It is easy to determine that all four of the axioms are true in a drawing of a triangle. A triangle is said to be a model of the three-point geometric axiomatic system described above.

We define a model to be any physical realization of the system where all the axioms hold.⁴ In the model of three-point geometry, you can see that the two theorems mentioned above are true. Two distinct lines do have a single point in common, and there are exactly three lines. The mathematical claim is that everything that

is true in the mathematical system must also be true in the model. The model may contain other truths that are not in the mathematical systems. For example, your model may have information concerning the length of edges. The axioms A1 to A4 have no information about length. Elements of a model often suggest new assumptions for the mathematical system. For example, if the lengths of edges were used to define distance between points, then theorems about distance could be stated.

It is important to realize that use of the word *model* above is different from the typical use. Here, the model may be bigger and more complex than the mathematical system. What makes it a model is that the axioms of the system hold in the realization.

How does a physical model, like that of a triangle for three-point geometry, establish the consistency of three-point geometry? The understanding is that contradictory statements cannot both be true of a physical model. That is, a physical state within a model cannot exist (true) and not exist (false) at the same time. If it appears this way, then it is really only a paradox, not a contradiction. For if there really were a contradiction, how could the model exist? Thus, if an axiomatic system is embedded in a model that actually exists, contradictions in the axiomatic system are assumed impossible. The ability to find a physical model of a mathematical system leads to assurances of consistency of the mathematical system.

What occurs when the consistency of a mathematical system is established by using models? By representing a mathematical system with a model, we are boldly transferring properties in creation to our thinking and reasoning.

Why should we believe that properties like consistency exist in creation? One answer is that God, through his providence, maintains the world around us in a consistent, predictable manner. “Natural Laws” hold from day to day, resulting in many experiences and observations that can be relied on. Even experiences and observations that are variable have variations that form reliable patterns. (Is anything without pattern?) Thus, the consistency reflected in laws and patterns is a reflection of God’s upholding hand in creation. As we seek to understand the world around us, we see the resulting consistency in creation and transfer

this property to our reasoning. This property, in turn, leads to the general belief in the reliability of mathematical knowledge. Thus, mathematical knowledge is ultimately based on God's providential and sustaining hand in creation.

2.2 Multiple Models

There may exist more than one model for an axiomatic system. For the three-point geometry discussed previously, one could have people as "points" and committees of two as "lines." The axioms would hold, as would the derived theorems. In our axioms, if one called lines *fum's* and points *fe's*, the results would still be present. They would just exist in terms of the new, undefined words: *fum* and *fe*.

Why bring this up? There are two reasons. The first is that having more than one model for a mathematical system leads us to the understanding that mathematics contains abstract concepts that depend on how you define the terms for mathematical objects. This diversity of models allows the same mathematics the ability to describe diverse phenomena and allows the transfer of a knowledge gain from one model to another. If mathematical reasoning were tightly associated with individual models, then generalizations to other models or situations would be impossible.

The movement of ideas between models and abstraction can be very subtle. This subtlety can be seen even in very simple things like the meaning of *two*. When *two* is used as an adjective, it is part of a model (i.e., "There are two people"). When *two* is used as a noun ($2+2=4$), you have moved into the realm of abstract concepts, into a mathematical system.⁵

The second reason to use *fum* and *fe* is to illustrate that reasoning and results in a mathematical system take place within the human mind and need not be associated with tangible things. It would be nice, if it were possible, to have a model of all abstract concepts and reasoning; however, this is not possible.

For example, our ability to count objects (using adjectives—*one*, *two*...—that establish nouns—*one*, *two*...—) and the fact that we can always count "one more" leads us to the concept of infinity. It's obviously impossible to physically model infinity. This impossibility makes an infinite set, like the set natural numbers which

are embedded in most mathematical systems, impossible to model. Mathematics quickly moves beyond the tangible world to the abstract world within our minds. As a result, it is not possible to determine if even "simple" mathematical systems are consistent, since there is no corresponding physical model to verify consistency.

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2.3 Summary of Section 2

We can summarize Section 2 as follows:

- Mathematical knowledge is based on

assumptions, and these are often related to models, since realization of the assumptions can give further ideas as to what assumptions to add or use.

- When a physical model satisfies the axioms of a mathematical system, it is assumed that everything that is true in the system is also true in the physical model.
- The belief that mathematical knowledge is certain is based on realizations of mathematical assumptions in physical models.
- There is an interplay between physical models and mathematical knowledge which leads us to believe in the reliability of mathematical knowledge and reasoning. I believe this to be a reflection of God's providential and sustaining hand in creation.
- Mathematical knowledge contains abstract concepts, many of which go beyond all possible physical models.

3 The Mathematical Modeling Process

We have examined the consistency of physical models in the development of mathematical systems and a few of the important assumptions that are implicit in these models. We now turn toward the thrust of mathematical modeling, that is, the use of mathematics to understand the physical world. The “mathematical world” and “physical world” are common to nearly all descriptions of the mathematical modeling process.⁶ While “model” in Section 2 referred to a physical realization of a mathematical system, “mathematical model” in this section refers to a mathematical system that describes aspects of the physical world. When I refer to the modeling process, what is meant is the development of the mathematical system.

3.1 Modeling an Iterative Process

The modeling process is an iteration scheme, where observations in the physical world lead to changes in a mathematical description of a phenomenon, and “observations” in the mathematical world (mathematical results) lead to exploring the physical world or to a changed understanding of the physical world. This sounds very much like the interplay between models and mathematical systems in Section 2. Let's consider

the modeling process in more detail. An outline of the process that could easily be found in a textbook on mathematical modeling is the following:

1. Start with a question about a problem that you would like to answer.
2. Isolate important parts of the problem.
3. Translate your observations into mathematics; that is, make your model.
4. “Do” the mathematics to see what you can discover in the mathematical world.
5. Translate your mathematical results into meaning for the physical world.
6. Validate your model against the physical world to see if it is reasonable.
 - (a) If the model is validated, ask what new truths it reveals about your original problem or some particular aspect of your problem.
 - (b) If your model is not validated, re-examine items 2, 3, and 4 and modify as needed.

The sixth step is iterated (repeated) until the modeler is satisfied that the model is sufficiently valid for the problem being addressed. While the mathematical modeling is being done, the “steps” above are generally all mixed up and appear in order only when a model is presented!

In the modeling process, step two is of extreme importance. It is, in essence, formulating how things interact and determining the primary and secondary influences within the problem. Rough pictures or caricatures of the problem are often drawn to help promote understanding during step two. Typically only important parts with primary effects are selected for incorporation into any mathematical model. If everything about the problem is incorporated into the mathematical model, the model generally becomes too complex to work with. I will say a bit more about the effect of simplification when we consider model validation in Section 3.3 below.

It is in step two that the modeler makes judgments as to what is important by isolating various portions of problem. Often these judgments will determine the outcomes. Consider inter-cellular calcium oscillations. Painting a very crude picture of the interacting parts, I'd describe the process at the cellular level as something like this: a hormone in extra-cellular medium attaches itself to a receptor on the cell surface, which in

turn triggers some membrane reactions that release molecules into the inter-cellular fluid. These molecules then interact with inter-cellular membrane receptors to cause the release of inter-cellular calcium, which then interacts with the membrane and other inter-cellular receptors in a manner which makes oscillations possible. This is a complicated process. Depending on your focus (cell membrane dynamics, inter-cellular membrane dynamics, inter-cellular calcium storage, or buffering) you can arrive at multiple models that have inter-cellular calcium levels that oscillate.⁷

The mathematical modeling process is an iterative scheme by which we try to refine our understanding of a physical-world phenomenon by translating assumptions about the phenomenon into mathematical language where mathematical results are discovered, which are then translated back to a physical world meaning.

Step three is where the building of the mathematical model occurs. The important parts of the problem are turned into assumptions. The assumptions are then translated (or embedded) into a mathematical framework of some sort (calculus, graph theory, algebra, etc.). The assumptions can be based on how individual elements in the physical

world interact. For example, what happens when one particle contacts another? Is momentum transferred? Do they react in some way? Or if the particles are people, is fear or disease transferred? Another option is that the assumptions could be based on an aggregate behavior. For example, in a fluid with chemicals, you might assume that it is well mixed and assume that a certain fraction of one chemical interacts with another over a short period of time. This is in contrast to particles interacting individually.

Experienced modelers generally have their favorite mathematical concepts or theories for translating the assumptions into mathematics. Some may prefer to use differential equations, others graph theory, others linear algebra. Obviously the model will differ, depending on the modeler's choices. Thus, any particular model reflects the strengths and creativity of the modeler. There is no one single model of anything.

Once the model is translated into a mathematical framework, the results within that framework can be applied to see what the mathematics reveals. This process often necessitates revisiting the understanding of the problem to make adjustments to the assumptions needed by the mathematics. With today's computers, it is quite common within the mathematical framework to do simulations as a way of suggesting possible results.

Once mathematical results are known, results are translated back to the world of the problem. For example, knowing the root of a function may translate into an optimal efficiency point for a process (e.g., how to tune a carburetor to achieve maximum fuel efficiency at a particular speed).

Validation of the model is, in essence, asking how well the model corresponds to observables within the framework of the original model. Does the mathematics describe the observables? Does it predict things that can be found in the original problem? If the model does not describe what you think is important, then modifications in the understanding of the original problem, the assumptions, or the translations are needed.

3.2 The Correspondence Assumption

The claim of Section 2 was that the certainty within mathematics is based on physical models, the realizations of axioms, or assumptions in the

physical world. The underlying assumption was that if the mathematical axioms hold of the model being investigated, then all of the results derived within the mathematical framework must also hold of the model. The modeling process is dependent on the veracity of this assumption. Assuming that there is a model that can be validated, the modeler, when presented with a model that is not validated, assumes that some of the assumptions embedded in the mathematical framework were incorrect.

The difference between the ideas of Section 2 and Section 3 is that in Section 2, we started with mathematical assumptions and tried to find a physical realization where those assumptions were valid. With the modeling process we try to find assumptions that fit a physical-world problem, assuming that a correspondence between a mathematical model and the physical-world problem is possible. The underlying assumption in both cases is that there is a tight correspondence between the problem realized in the physical world and mathematical reasoning in the mathematical world. The mathematical modeler would like to believe that the knowledge gained from the mathematics helps us understand the complexities of the original problem. That is, if the assumptions of the mathematical world and the physical realization match, then mathematical reasoning will translate into true knowledge of the physical-world problem. This is called the *correspondence assumption*.

3.3 “Wrong” Models

Let’s step back. Consider step two of the modeling process again: “Isolate important parts of the problem.” The main objective here is to reduce the complexity of the problem. A second, usually unstated, objective is to deal with a limited understanding of the original problem. If we knew exactly how things worked in the physical world, why would we be using mathematics to gain more understanding? We might be using mathematics to extol the beautiful mathematical nature of the world; however, we generally have a finite understanding of the process that is being modeled, and we seek better understanding of that process.

The question arises as to the quality of results that are based on approximated or isolated aspects of the physical-world problem. In particular, there

are parts of the physical-world problem that will not be part of the mathematical model due to simplifying assumptions. Further, there may even be assumptions which are not exactly correct. It is clear that inherent in the mathematical modeling process, errors and approximations abound! What can be learned from a wrong model?

Consider the following: A “true” model is one that can be perfectly validated, that is, where all information in the mathematical model is present in the physical model and vice versa. Another way to say this is that there is no disagreement between observables and mathematical results. It is always the case that a true model can be arrived at by taking a wrong model (some validation fails) and adding corrections.⁸ In symbols this looks like

$$T = W + C$$

where T represents a true model, W the wrong model, and C the correction. Factoring out W leads to

$$T = W(1 + C/W).$$

If C is very small compared to W , that is C/W is very close to zero, then the wrong model can be considered very close to the true model. That is

$$T \approx W.$$

This simply implies that even if we assume that all models are wrong in some regards (almost all modelers will agree with this), the wrong model will behave in a similar fashion to the true model when we are close enough. Meerschaert, in his book *Mathematical Modeling*,⁹ calls this property of models *robustness*. Small changes in the mathematical model don’t change the behavior of the model, and if we are close enough to the true model, the true model’s behavior is describable by the wrong model.

In essence the understanding gained from “wrong” models is often close enough to the “true” model that we are not faced with a world that seems unintelligible and devoid of pattern. Rather, we are faced with a world that can be understood. How close is close enough? This is a decision of the modeler and the problem being modeled and, thus, depends on the level of detail a modeler requires.

3.4 Summary of Section 3

We can summarize Section 3 as follows:

- The mathematical modeling process is an iterative scheme by which we try to refine our understanding of a physical-world phenomenon by translating assumptions about the phenomenon into mathematical language where mathematical results are discovered, which are then translated back to a physical world meaning.
- More than one model can be derived for any given phenomenon. The type of model and details of a model are dependent on the choices the modeler makes.
- If the modeling assumptions are close to true in the physical world, then mathematical results of the model will transfer to knowledge about the physical world.
- Mathematical models of physical world phenomena are inherently incorrect, but this incorrectness generally does not keep them from being useful.

4 Limitations of Mathematical Models

From the wide-spread use of mathematical models, it is evident that Western culture has placed a significant amount of trust in the mathematical modeling process. Often this trust is bolstered by achievements in areas like engineering, medical imaging, automobile designing, etc. Mathematical modeling has had less success in areas such as ecology, human behavior, long-term weather forecasting, and in systems where predictability is limited to statistical descriptions. In these areas, however, there seems to be an increasing reliability on the predictive power of mathematical modeling and simulation, as the modeling process iterates closer to the “true” model. Is it really possible to iterate toward the true model?

In sections 2 and 3, the focus was on the process of modeling and on assumptions concerning the physical world that allow the building of mathematical models. Although there are many directions that one could follow when discussing the limitations of mathematical models, we will limit the discussion to two observations concerning the iterative nature of the modeling process. These observations seek to reduce the magnitude of corrections in a model and include an observation

on the interactive nature of modeling.

The iterative nature of the modeling process is essential, but each iteration is limited. Each modeling iteration allows the modeler to integrate new information into a model. This integration is often done by changing assumptions or refining ranges of parameters within an existing model. However, for any given set of assumptions in a mathematical model, there is a limit to the information that can be contained in the resulting mathematical model. This limit may be unknown and large, but a limit still exists. The real limitation is that it is a subjective choice as to how many iterations are needed before useful information is attained. That is, when is the wrong model close enough to the true model?

An analogy to linear algebra may be helpful. In linear algebra a set of vectors will span a vector space. Many interesting things may be waiting to be discovered in the vector space; however, to move beyond that particular span of vectors, say into a new dimension, one must add other vectors to the original set. Axioms are like the starting set of vectors. There may be many fruitful results that are gained, but to move beyond the span of the axioms in other directions for better approximations, one must add new axioms or change existing ones in order to explore new dimensions and arrive at better approximations. The problem is when to stop tweaking the axioms. The end of the iterative process depends on when the modeler is satisfied with the validation process.

Another limitation inherent in the process of mathematical modeling is the adjustment of parameter values that occurs as new information is discovered about the problem being modeled. A parameter is some unknown aspect contained in an assumption of the model. Often this is a numerical value that is needed if one is to make the assumption concrete.

Parameter values used in models usually apply only to limited situations. For example, the newspaper of the town I live in reports that over the last 10 years the town has been growing at about seven percent per year. If a modeler on the town council used this assumption for planning, it would probably be reliable for a few years into the future, but if he/she were to plan for 30 years into the future, the assumption would not be valid.

There are ways around this problem. For

example, we could assume that the percentage rate of growth is given by an unknown function of time, $r(t)$, for which we know only characteristics but not specific details. New results could be gained from this modification, but again the assumption on the characteristics of $r(t)$ would limit the knowledge gained for the model. General trends may be gained, but specific details are almost always lost. This gain/loss illustrates a general property of modeling, which sounds obvious but needs to be stated: the more general a model is, the less specific it can be. Models that try to predict general patterns are often called *qualitative*, while models that predict detailed information, like the breaking point of a beam or the length of time beach restorations will last, are called *quantitative*.

Our final observation is on the interactive nature of the modeling process. The ability of the modeler to choose the axioms of a model clearly indicates that mathematical models are a result of human understanding. Since the process relies on human understanding, it is not possible a priori to determine which assumptions require modification, though after the modification, it almost always seems as if the modification was just waiting to happen. This is like the statistician saying that the most probable thing to happen is what has happened. Essentially, it is not possible to anticipate how axioms or parameters within a model need to be changed without comparing the model with new data. It is like wandering around in the mathematical world with a light shining in from the physical world. What you “see” depends on your frame of mind and previous experiences. Changes in a model during the modeling process are in response to human observations, insight, and judgment.

4.1 Summary of Section 4

We can summarize Section 4 as follows:

- Mathematical models are subject to the limitation that the axiom system imposes. That is, the results possible are found only within the span of the axioms. It is not possible to predict beyond the assumption of a model.
- The more general a mathematical model is, the less specific its results will be. This is often phrased as a distinction in model type: *qualitative* vs *quantitative*.

- The certainty that a mathematical model provides is limited to the certainty of human observations, insight, and judgment.

5 Conclusion

The meaning of mathematical knowledge ultimately comes from physical realizations of mathematical assumptions and extensions of observations (e.g., “We can always count one more.”). Confidence in the correspondence assumption and human insight is the foundational element of mathematical modeling. The underlying

Christians are called by the Bible to care for and develop creation. This means that we need to understand the world around us. One way to do this is to formulate mathematical models. Thus, a Christian approach to mathematical modeling is to understand the limitations of mathematical models and the modeling process.

assumption is that with the correct mathematical assumptions, found through human insight, it is possible to build a mathematical model that will give the modeler further insight into the problem being modeled. The process of mathematical modeling embodies interplay among the modeler’s

knowledge, observations, and assumptions.

With this interplay in mind, we now ask the following question: When the results of mathematical models are used to predict outcomes or are otherwise relied on, what are we trusting or putting faith in? Here are three of the larger places where trust is placed:

- The modeler's choices and abilities (or the abilities of programmers if computer simulations are used).
- The validity of the assumptions worked into the model.
- The correspondence assumptions.

It should be clear that the first two items are prone to human limitation, which may result in large or small errors. When a result of a mathematical model is important, potential error is mitigated by using multiple people to check models and assumptions and by comparing results of multiple models. The third item, which underpins the others, is an assumption that I believe relies on God's providential and sustaining hand in creation.

The sin of the mathematical modeler is to ignore the basis for the last item and redirect glory that is due to God toward human reasoning and observation in the first two items. This is particularly true with successful mathematical models. How should the Christian modeler who professes ultimate faith and trust in God respond?

Christians are called by the Bible to care for and develop creation. This means that we need to understand the world around us. One way to do this is to formulate mathematical models. Thus, a Christian approach to mathematical modeling is to understand the limitations of mathematical models and the modeling process. Also, when applying results, we should act in humility, realizing that we are responding to human knowledge gained from human observations and human reasoning. If mathematical models give results that don't correspond with observations, then we should recognize the failings and adapt, not necessarily letting the current models or thinking rule our responses. This is not an exclusive response, limited to Christians. When mathematical models do correspond with observations or reveal previously unknown structure, the Christian should give glory to God and respond in a manner that is believed to honor God further and care for His creation.

Thus, unlike the theme of the novel *Polar Shift*, where knowledge of mathematical models gives power to manipulate creation for one's own purposes, Christians should recognize that knowledge of mathematical models demonstrates God's glory by revealing structure otherwise hidden, and their response should be to further glorify God in actions and thoughts.

Endnotes

1. Paul Kemprecos and Clive Cussler, *Polar Shift (NUMA Files)* (NY: G.P. Putnam's Sons, 2005).
2. Morris Kline, *Mathematics and the Search for Knowledge* (Oxford: Oxford University Press, 2005).
3. A paradox is a group of true statements that appear to be contradictory. The appearance of a paradox generally results from lack of complete understanding. With better understanding the paradoxical elements can be shown not to contradict each other.
4. This definition of model is more restrictive than that used in Model Theory, where a structure that gives meaning to the sentences of a formal language is called a model for the language and there is no "physical" realization requirement. Having a model that exists within mathematics system, A, for a mathematical system, B, provides certainty of properties in B on the assumption of a those properties in A. This leads to reasoning totally contained only in mathematics. Using "physical" realizations provides evidence for properties of a mathematical system from outside mathematics.
5. See Reuben Hersh, *What is Mathematics, Really?* (Oxford: Oxford University Press, 1997), 15.
6. A note of clarification: model means two different things in section 2 and section 3.
7. See the following for the three models: M.J. Berridge, "Cytoplasmic calcium oscillations: a two pool model," in *Cell Calcium* 12 (March 1991): 63-72; K.S. Cuthbertson and T.R. Chay, "Modelling receptor controlled intracellular calcium oscillators," in *Cell Calcium*, 12 (Feb-Mar, 1991): 97-109; and T. Meyer and L. Stryer, "Calcium spiking," in *Annual Review of Biophysics/ Biophysical Chemistry* 20 (1991): 153-174.
8. This illustration was heard in a presentation by Robert Padoff, of The Shodor Education Foundation, at an SC09 Education Workshop, held at Calvin College, May 2009.
9. Mark Meerschaert, *Mathematical Modeling* (Academic Press, 1993), 14.